Strong Motion Calculations

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Specification

Part No. SWA-RFC-STMN

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Contents

1 Introduction

Güralp digitisers are capable of performing certain real-time strong motion calculations. This document discusses both the calculations and the representation of their results, and defines a new type of GCF block (the strong motion block, stream ID ending SM) which is used to transmit the results to a data acquisition system.

An initial set of calculations was added to the CMG-DM24-mk3 model digitiser. These calculations are hard-coded and referred to as the basic calculations, discussed in chapter 2.

It has become apparent that there are a wide variety of different calculations which will be required, and each of these calculations may be tuned to a specific installation. This has led to the development of a secondary DSP module for the CMG-CD24 model digitiser, which is capable of being programmed by an attached CMG-EAM and whose purpose is solely to compute further strong motion calculations.

2 Basic Calculations

In this document, y_t represents the most recent sample from the accelerometer. The previous sample is y_{t-1}, and so on. These calculations are performed on the low-latency 20sps data, although in future this might be set to another output tap.

2.1 Windowed MMA (minimum, maximum, average)

This calculation finds the minimum, maximum and average (mean) samples within the last 10 seconds (configurable), and reports these values every second. The minimum and maximum are found over the last second of data; the average over the last 10 seconds.

$$
\begin{aligned}
m_t &= min\left(y_{t-19}, ..., y_t\right) \\
M_t &= max\left(y_{t-19}, ..., y_t\right) \\
\bar{y_t} &= \frac{\sum_{\tau=0}^{199} y_{t-\tau}}{200} \\
t &= 20, 40, 60...
\end{aligned}
$$

The average value $\bar{y_t}$ is taken to be the DC component and is used in the PGA and RMS calculations. The minimum m_t and maximum M_t are used in the PGA calculation.

2.2 PGA (peak ground acceleration)

The PGA is simply the largest acceleration recorded within the time window being covered by the MMA calculation, converted to ground units. It is output once a second, along with the MMA.

The DC component $\bar{y}_t$ is first subtracted from the minimum and maximum values, m_t and M_t. The result with the largest magnitude is taken to be the PGA. This value is then converted from digital counts into ground units.

$$P_t = \frac{100.max\left(\left|m_t - \bar{y}_t\right|, \left|M_t - \bar{y}_t\right|\right).v_c}{v_g}$$

v_c is the volts per count calibration value for the digitiser channel. v_g is the volts per ground unit (ms^{-2}) calibration value for the instrument channel. The factor of 10 is so that the output unit is in gal (cms^{-2}).

2.3 RMS (root mean square)

The RMS value is computed every second based on one second of data and the average $\bar{y}_t$ from the MMA time window. It is defined as:

$$R_t = \sqrt{\frac{\sum_{\tau=0}^{19}\left(y_{t-\tau} - \bar{y}_t\right)^2}{20}}$$

2.4 Similarity Index

The similarity index value is computed every second, and uses the last one second of data. It is defined as:

$$C_t^{ij} = \frac{\sum_{\tau=0}^{19}\left(\left(y_{t-\tau}^i - \bar{y}_t^i\right).\left(z_{t-\tau}^j - \bar{z}_t^j\right)\right)}{\sqrt{\sum_{\tau=0}^{19}\left(y_{t-\tau}^i - \bar{y}_t^i\right)^2 . \sum_{\tau=0}^{19}\left(z_{t-\tau}^j - \bar{z}_t^j\right)^2}}$$

Here i and j refer to channels, and C_t^{ij} is the similarity index between the ith channel of sensor y and the jth channel of sensor z. For this application we will only want to compute the similarity between the vertical channels of two different instruments plugged into the same DM24.

The calculation should give an answer close to 1 or -1, regardless of calibration, which would indicate a healthy system. Any other value would indicate a potential problem with one or both sensors.

2.5 Spectral Intensity

The original reference for development of the SI algorithm is the paper *Earthquake Sensor* (T. Yanada et al.). Some of the equations below were derived from this paper by Paul Minchinton.

Definition of SI

We define the following variables:

$$\begin{aligned} y(t) &\rightarrow \text{acceleration} \\ h &\rightarrow \text{damping ratio} \\ T &\rightarrow \text{natural period of oscillation of building} \\ v(T,h,t) &\rightarrow \text{pseudo-velocity} \\ Sv(T,h,t) &\rightarrow \text{maximum pseudo-velocity in time window} \end{aligned}$$

The acceleration $y(t)$ is the input to the system. It is the waveform from the sensor, and its unit will be ms^{-2}.

The pseudo-velocity is the theoretical velocity a structure would attain if we assume that it is a second order system with natural period T and damping ratio h. Throughout the calculations, we assume a fixed value of h, which leaves v (and Sv) as a function of T and t.

To compute the spectral intensity at time t, we find the average value of Sv over a range of natural periods from 0.1 to 2.5s:

$$SI(t) = \frac{1}{2.4} \int_{0.1}^{2.5} Sv(T,h,t)dT$$

Since we are using a digital system, we have discrete-time functions rather than continuous. The notation x_t denotes the sample of the function $x(t)$ at time instant t (where $t = 0, 1, 2...$).

Computation of pseudo-velocity

The pseudo-velocity is computed using the following equation:

$$\frac{v_t}{y_t} = \frac{A_0 + A_1 z^{-1} + A_2 z^{-2}}{1 + B_1 z^{-1} + B_2 z^{-2}}$$

The parameters A and B (which depend on h and T) are defined as:

$$\begin{aligned}
\omega &= \frac{2\pi}{T} \\
\omega_d &= \omega\sqrt{1-h^2} \\
E &= e^{-h\omega\Delta t} \\
c &= \cos\omega_d\Delta t \\
s &= \sin\omega_d\Delta t \\
A_{11} &= E\left(c + \frac{h\omega}{\omega_d}s\right) \\
A_{12} &= E\frac{1}{\omega_d}s \\
A_{21} &= -E\frac{\omega^2}{\omega_d}s \\
A_{22} &= E\left(c - \frac{h\omega}{\omega_d}s\right) \\
B_{11} &= E\left[\left(\frac{1}{\omega^2} + \frac{2h}{\omega^3\Delta t}\right)c - \left(\frac{h}{\omega\omega_d} + \frac{1-2h^2}{\omega^2\omega_d\Delta t}\right)s\right] - \frac{2h}{\omega^3\Delta t} \\
B_{12} &= E\left[-\frac{2h}{\omega^3\Delta t}c + \frac{1-2h^2}{\omega^2\omega_d\Delta t}s\right] - \frac{1}{\omega^2} + \frac{2h}{\omega^3\Delta t} \\
B_{21} &= E\left[-\frac{1}{\omega^2\Delta t}c + \left(\frac{h}{\omega\omega_d\Delta t} + \frac{1}{\omega_d}\right)s\right] + \frac{1}{\omega^2\Delta t} \\
B_{22} &= E\left[\frac{1}{\omega^2\Delta t}c + \frac{h}{\omega\omega_d}s\right] - \frac{1}{\omega^2\Delta t} \\
A_0 &= B_{22} \\
A_1 &= A_{21}B_{12} + B_{21} - A_{11}B_{22} \\
A_2 &= A_{21}B_{11} - A_{11}B_{21} \\
B_1 &= -A_{11} - A_{22} \\
B_2 &= A_{11}A_{22} - A_{21}A_{12}
\end{aligned}$$

The pseudo-velocity is the theoretical velocity a structure with damping ratio h, and natural period of oscillation T would achieve when forced by an acceleration

$y(t)$. This is correlated with the amount of energy absorbed by the structure (and, thus, with the amount of damage it would theoretically sustain). We must compute a range of pseudo-velocities and then integrate (average) over them in order to compute the total energy absorbed (i.e. the *spectral intensity*).

Computation of SI along one axis

The input waveform $y(t)$ is sampled at 20Hz. The pseudo-velocity is computed (for each sample of y_t) at seven different values of T (0.1, 0.4, 0.7, 1.0, 1.5, 2.0 and 2.5s), thus leaving it as a two-dimensional array (time as one dimension and T as other; a $20n \times 7$ array, where n is the number of seconds). The notation $v_{t,0}...v_{t,6}$ will be used to refer to elements within this array.

The maximum pseudo-velocity is computed once each second. $Sv_{t,n}$ is the maximum value of $|v_{t,n}|$ (the magnitude of the pseudo-velocity) over a sliding time window. For example, at $t = 400$ (20s), with a 10s window:

$$Sv_{400,3} = max\,|v_{400,3}, v_{399,3}, ..., v_{201,3}|$$

Each second, there will be seven values of Sv calculated, and one SI result. The SI is simply the average of these:

$$\begin{aligned} SI_t &= \frac{1}{7}\sum_{n=0}^{6} Sv_{t,n} \\ \text{for } t &= 0, 20, 40, 60, ... \end{aligned}$$

3 Outputs

3.1 Components

Each output may apply to one or more component. The components are as follows:

Z,N,E — The three components of a triaxial instrument.

H — The resultant in the horizontal (N,E) components, $\sqrt{N^2 + E^2}$.

A — The resultant in all three components, $\sqrt{Z^2 + N^2 + E^2}$.

(I) — Output applies to instrument as a whole.

Resultants are computed as a simple vector length based on the individual component results, regardless of the underlying calculation.

3.2 Basic Calculation Outputs

Output formats are summarised in table 3.1. Currently, all basic calculation outputs are floating point.

Calculation	Format	Component	Unit
Windowed MMA	float,float,float	Z,N,E,H,A	gal
PGA	float	Z,N,E,H,A	gal
RMS	float	Z,N,E,H,A	gal
Cross-correlation	float	(I)	(unitless)
SI	float	Z,N,E,H,A	$kine$

Table 3.1: Basic calculation output details.

3.3 Advanced Calculation Outputs

Using the advanced calculation architecture, the calculations are programmed at runtime. Multiple calculations may be run in parallel, with the results being transmitted together as parts of the same SM block. The two currently-defined output formats are a stream of integers or a stream of floating point numbers. The stream may have one or more samples.

SM blocks are transmitted with a 1-second granularity. A calculation which had an output rate of 20sps would therefore have a stream of 20 samples within each SM block. If all the results do not fit into a single SM block, further SM blocks with the same timestamp may be transmitted.

Sample streams are preceded in an SM block by a type and a tag. The tag can be used to identify the calculation from which the stream resulted. This will then be associated by the device that programmed the DSP initially with the true meaning/channel name of the calculation.

4 Packet Format

The packet is a GCF block with a stream ID ending in 'SM'. The other header details are the same as a normal ('00') status block. This packet is emitted each second.

A tagged packet format (similar to unified status packets) is employed to allow for forwards and backwards compatibility. Each result is preceded by a tag stating what the result is for. The tag is a 32-bit integer, broken down as follows:

31 30 29 28 27 26 25 24 23 22 21 20 19 18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 0

Type	Tag	Component	Length

Type The type code. See table 4.1 for assignments.

Tag The tag code. If the type is 0 (basic calculations), table 4.2 lists the assignments. If the type is advanced calculation, then the tags are arbitrary and correspond to the arrangement of calculations on the DSP.

Component The component code. See table 4.3 for assignments. Uses the same system as channel status for unified status packets.

Length The length of the result, in 32-bit data words (00 = 1 word, 01 = 2 words, ...).

Scalar values are given as 32-bit IEEE floating point objects, and written in big-endian (network) byte order. Integers are also written in big-endian byte order.

Type code	Description	Result format
0x0	Basic calculation result	As per table 4.2.
0x1	Advanced calculation result	Stream of floating point numbers.
0x2	Advanced calculation result	Stream of integers.

Table 4.1: Type codes.

Calculation	Tag code	Length code	Result format
Windowed MMA	0x000	0x02	float,float,float
PGA	0x001	0x00	float
RMS	0x002	0x00	float
Cross-correlation	0x003	0x00	float
SI	0x004	0x00	float

Table 4.2: Tag codes.

Component	Code
Channel '0' of instrument 0	0x00
Channel '1' of instrument 0	0x01
...	
Channel 'A' of instrument 0	0x0A
...	
Channel 'Z' of instrument 0	0x23
Horizontal resultant of instrument 0	0x24
3D resultant of instrument 0	0x25
Channel '0' of instrument 1	0x80
...	
Channel 'Z' of instrument 1	0xA3
Horizontal resultant of instrument 1	0xA4
3D resultant of instrument 1	0xA5
Whole instrument	0xFF

Table 4.3: Component codes.

If the digitiser is in serial2 mode (i.e. appearing as two digitisers with different IDs), then the packet should be split so that the results for the second instrument can be associated with the second instrument's data. In this case, any tags which have a component code of 0x80 or greater will be moved into a second SM packet (using the second ID), and renumbered to appear as instrument 0. Anything which affects the instrument as a whole (component code 0xFF) will simply be duplicated.

5 Revision history

2009-01-23 C Use standard Güralp formatting. Add introductory section. Add result type code. Add text for CD24 dual-DSP mode ("advanced calculations").

2008-11-18 B Cross-correlation renamed to Similarity Index. Added section on Spectral Intensity. Minor changes to packet format.

2007-09-26 A Initial release.